AHA Algebra Proficiency Exam Study Guide

Key

1. At First Street Elementary School, about 18% of the 610 students ride bicycles to school.

a. Set up a proportion and solve it to find out how many students ride bicycles to school?

110 $\frac{18}{100} = \frac{x}{610}$ $18(610) = 100x$ $10980 = 100x$ $\frac{10980}{100} = x \approx 109.8$ (110 ✱)

b. The following year, First Street Elementary has 650 students. What is the percent increase or decrease?

6.55% ↑ $\frac{\text{change}}{\text{orig}}$ $\frac{650-610}{610} \Rightarrow \frac{40}{610} = .0655 \rightarrow 6.55\% \uparrow$ ✱

2. Mandy is buying a new graphing calculator that is marked as costing $89.99. She has a coupon for a 15% discount and knows she will be charge a 5% sales tax. How much will she pay for the calculator?

$80.31 15% off means paying 100% − 15% = 85%

.85(89.99) = 76.49 ← sale price

5% tax + 100% = 105% 76.49 → 1.05(76.49) = $80.31 ✱

3. Asanji took a trip to Mexico. Upon leaving he decided to convert all of his Pesos back into dollars. How many dollars did he receive if he exchanged 42.7 Pesos at a rate of $5.30 = 11.1 Pesos?

$20.38 $\frac{\$5.30}{11.1 \text{ pesos}} = \frac{x}{42.7}$ $5.30(42.7) = 11.1x$

$226.31 = 11.1x \rightarrow \$20.38 = x$ ✱

4. A concession stand is selling hot dogs and sodas. One group purchases 4 hot dogs and 5 sodas and their total comes to $14.55. Another group purchases 6 hot dogs and 8 sodas and their total comes to $22.40.

hot dog = $2.20

soda = $1.15

a. Write a system of equations to represent this situation.

$4h + 5s = 14.55$

$6h + 8s = 22.40$

b. Solve this system of equations to find the price of a hot dog and the price of a soda.

$3(4h + 5s = 14.55) \rightarrow 12h + 15s = 43.65$

$-2(6h + 8s = 22.40) \rightarrow -12h - 16s = -44.80$

$-1s = -1.15$

soda = $1.15 ✱

$4h + 5(1.15) = 14.55$

$4h + 5.75 = 14.55$

$4h = 8.80 \rightarrow h = \$2.20$ ✱

5. a. A new car depreciates at 18% per year. If it costs $20,000 when new, write an equation that models how much the car is worth after *t* years.

6 90424u

$100 - 18 = 82\%$

$V = 20{,}000(.82)^t$

b. Use your equation to find how much the car is worth 4 years after it was purchased.

$V = 20{,}000(.82)^4$

$= 9042.44$ ✱

Simplify each expression.

6. $2x + 9 - 8x + 13$

$-6x + 22$

7. $3x^2(2x^3 - 7y)$

$6x^5 - 21x^2y$

8. $(2x - 5)(3x + 7)$

$6x^2 + 14x - 15x - 35 \rightarrow 6x^2 - 1x - 35$

9. $(-x^2 - 4x + 1) + (x^2 + 2x + 3)$

$-2x + 4$

10. $(2q^3 + 6q^2 + 9q) - (q^3 + 11q^2 + 3)$

$1q^3 - 5q^2 + 9q - 3$

11. $6x^3(4xy) \rightarrow 24x^4y$

12. $-2k^2(k^4 - 4k^3)$

$-2k^6 + 8k^5$

13. $(2y + 5)(3y - 3)$ $6y^2 - 6y + 15y - 15$

$6y^2 + 9y - 15$

14. $(x + 6)(x - 6)$

$x^2 - 36$

15. $(m - 8)^2$ $m^2 - 16m + 64$

16. $(4x + 6)^2$

$16x^2 + 48x + 36$

17. $(4a - 3)(4a + 3)$ $16a^2 - 9$

18. $(2x + 2y)^2$

$4x^2 + 8xy + 4y^2$

19. $(3x - 4)(4x^2 + 2x - 7)$ $12x^3 + 6x^2 - 21x - 16x^2 - 8x + 28$

$12x^3 - 10x^2 - 29x + 28$

20. $\frac{3x}{2} \div \frac{4}{1} \rightarrow \frac{3x}{2} \cdot \frac{1}{4} = \frac{3x}{8}$

21. $-\frac{12x^2}{3} \cdot \frac{2y}{7} \rightarrow -\frac{8x^2y}{7}$

22. $4(x + 3) + 6(2x - 8)$

$4x + 12 + 12x - 48 \rightarrow 16x - 36$

23. $c^2(3c - 8) + c(4c^2 + 2c - 7)$

$3c^3 - 8c^2 + 4c^3 + 2c^2 - 7c \rightarrow 7c^3 - 6c^2 - 7c$

24. $2(xy^2z)^5$

$2(x^5y^{10}z^5) \rightarrow 2x^5y^{10}z^5$

25. $[(4a^3b)(-3ab)^2]^4$

$[4a^3b(9a^2b^2)]^4 \rightarrow [36a^5b^3]^4 \rightarrow 36^4a^{20}b^{12}$

26. $\frac{2de^5}{3de}$ $\frac{2e^4}{3}$

27. $\frac{85x^4y^{-3}z}{15x^2yz^{-1}} \rightarrow \frac{17x^2zz}{1yy^3} \rightarrow \frac{17x^2z^2}{y^4}$

Solve each equation, inequality, or system of equations. Show your work!

28 $-28 = 10w - 3w$

$\frac{-28}{7} = \frac{7w}{7} \rightarrow w = -4$ ★

29. $\frac{d}{5} + 1 = 7$ $-1 \quad -1$

$\frac{d}{5} = \frac{6}{1}$

$d = 30$ ★

30. $\frac{9}{4}y - 2 = 25$ $+2 \quad +2$

$\frac{9}{4}y = 27 \cdot \frac{4}{9}$

$y = \frac{3}{1}\cdot\frac{4}{1} = 12$ ★

31. $7(h + 3) + 4 = -3$

$7h + 21 + 4 = -3$

$7h + 25 = -3$

$7h = -28$

$h = -4$ ★

32. $6 - 11x = 7x - 12$

$6 = 18x - 12$

$18 = 18x$

$1 = x$ ★

33. $2y + 5 = 3(4y - 5)$

$2y + 5 = 12y - 15$

$5 = 10y - 15$

$20 = 10y$

$2 = y$ ★

Cross mult

34. $\frac{t}{65} = \frac{5}{13}$

$13t = 325$

$\frac{13t}{13} = \frac{325}{13}$

$t = 25$ ★

35. $\frac{m+18}{m} = \frac{5}{2}$

$2(m+18) = 5m$

$2m + 36 = 5m$

$36 = 3m$

$3 = m$ ★

36. $\frac{6}{x+4} = \frac{12}{5x-13}$

$6(5x-13) = 12(x+4)$

$30x - 65 = 12x + 48$

$18x - 65 = 48$

$18x = 113$

$x = 6.28$

37. $-\frac{1}{2}x - 9 < 3$

$-\frac{2}{1}\left(-\frac{1}{2}\right)x < 12\left(-\frac{2}{1}\right)$

$x > -24$

flip ineq. b/c ÷ neg.

38. $5x + 6 > -2x + 13$

$7x + 6 > 13$

$7x > 7$

$x > 1$

no flip

39. $a^2 - 8a - 20 = 0$

$(a - 10)(a + 2) = 0$

$a - 10 = 0$ | $a + 2 = 0$

$a = 10$ | $a = -2$ ★

40. $h^2 - 10h + 32 = 8$

$-8 \quad -8$

$h^2 - 10h + 24 = 0$

$(h - 6)(h - 4) = 0$

$h - 6 = 0$ | $h - 4 = 0$

$h = 6$ | $h = 4$

41. $(5m + 4)(m - 10) = 0$

$5m + 4 = 0$ | $m - 10 = 0$

$5m = -4$ | $+10 \quad +10$

$m = -\frac{4}{5}$ | $m = 10$

42. $x^2 + 12x = 7$ ← must get = 0

$x^2 + 12x - 7 = 0$

not possible to factor

$\frac{-12 \pm \sqrt{12^2 - 4(1)(-7)}}{2} =$

$\frac{-12 \pm \sqrt{172}}{2} = \frac{-12 \pm 2\sqrt{43}}{2}$

$= -6 \pm \sqrt{43}$ ★

43. $8x^2 - 22x - 21 = 0$

$(2x - 7)(4x + 3) = 0$

$2x - 7 = 0$ | $4x + 3 = 0$

$x = \frac{7}{2}$ | $x = -\frac{3}{4}$

44. $3x + 4 - 5(-3x + 4) \geq 6 + 2(11x - 4)$

$3x + 4 + 15x - 20 \geq 6 + 22x - 8$

$18x - 16 \geq 22x - 2$

$-14 \geq 4x$

$-\frac{14}{4} \geq x$

$-\frac{7}{2} \geq x$ ★

45. $y = 5x - 11$

$y = -2x + 10$

use Substitution!

$5x - 11 = -2x + 10$

$7x = 21$

$x = 3$ ★

$y = 5(3) - 11$

$15 - 11$

$y = 4$ ★

$(3, 4)$ ★

46. $y = x - 4$

$x + 2y = 1$

use substitution

$x + 2(x - 4) = 1$

$x + 2x - 8 = 1$

$3x = 9$

$x = 3$ ★

$y = 3 - 4$

$y = -1$ ★

$(3, -1)$ ★

47. $3x + 2y = 4$ by elimination

$-3(x - 5y = -27)$

$3x + 2y = 4$

$-3x + 15y = 81$

$17y = 85$

$y = 5$ ★

$3x + 2(5) = 4$

$3x + 10 = 4$

$3x = -6$

$x = -2$ ★

$(-2, 5)$ ★

Simplify each expression. The radicals should be written in simplified radical form.

48. $\sqrt{144} = 12$

49. $\sqrt{32} = \sqrt{16 \cdot 2} = 4\sqrt{2}$

50. $\sqrt{250} = \sqrt{25 \cdot 10} = 5\sqrt{10}$

51. $\sqrt{54}$ $\sqrt{9 \cdot 6}$ $= 3\sqrt{6}$

52. $7\sqrt{5} \cdot 4\sqrt{2}$ $28\sqrt{10}$

53. $6\sqrt{3} + 5\sqrt{3}$ $11\sqrt{3}$

54. $(4\sqrt{5})^2$ $4\sqrt{5} \cdot 4\sqrt{5}$ $16 \cdot \sqrt{25}$ $16 \cdot 5$ 80 ★

55. $(\sqrt{3}+2)(\sqrt{3}-5)$ $\sqrt{9} - 5\sqrt{3} + 2\sqrt{3} - 10$ $3 - 3\sqrt{3} - 10$ $-7 - 3\sqrt{3}$

56. $2\sqrt{3} + 7\sqrt{5}$ done!

Solve for x. Exact answers ONLY.

57. $3x = (4\sqrt{2})(3\sqrt{2})$ $3x = 12\sqrt{4} \rightarrow 3x = 12 \cdot 2$ $3x = 24$ $x = 8$ ★

58. $\sqrt{2}(3\sqrt{2} + x) = 6 - 4\sqrt{2}$ $3\sqrt{4} + x\sqrt{2} = 6 - 4\sqrt{2}$ $6 + x\sqrt{2} = 6 - 4\sqrt{2}$ $x = -4$ ★

59. $\dfrac{1}{\sqrt{(x+5)^2}} = 4$ $\dfrac{1}{x+5} = 4$ $1 = 4x + 20$ $-19 = 4x$ $\dfrac{-19}{4} = x$ ★

60. $(2x-5)^2 = 12$ $(2x-5)(2x-5) = 12$ $4x^2 - 20x + 25 = 12$ $4x^2 - 20x + 13 = 0$ OR $2x - 5 = \pm 2\sqrt{3}$ must $= 0$ $2x = 5 \pm 2\sqrt{3}$ $x = \dfrac{5 \pm 2\sqrt{3}}{2}$

$x = \dfrac{20 \pm \sqrt{401 - 4(4)(13)}}{2(4)} = \dfrac{20 \pm \sqrt{192}}{8} \rightarrow \dfrac{20 \pm \sqrt{64 \cdot 3}}{8}$ $\dfrac{20 \pm 8\sqrt{3}}{8}$ ★ $\dfrac{5 \pm 2\sqrt{3}}{2}$

Factor each expression.

61. $6x^3 - 15x$ $3x(2x^2 - 5)$ ★

62. $x^2 + 5x - 6$ $(x+6)(x-1)$ ★

63. $-9x^2y - 3x$ $-3x(3xy + 1)$ ★

64. $x^2 + 8x + 12$ $(x+6)(x+2)$ ★

$(4x+3)(1x+5)$ ★

65. $x^2 - 8x + 16$ $(x-4)(x-4)$ ★ $(x-4)^2$ ★

$(3x+2)(x+5)$ ★

66. $3x^2 + 7x - 6$ $(3x-2)(x+3)$ ★

$-1(2y+15)(y-1)$ ★ $-1(2y^2 - 13y + 15)$

67. $4x^2 + 23x + 15$

68. $3x^2 + 17x + 10$

69. $-2y^2 + 13y - 15$

70. $5n^2 - 5n - 60$

$5(n^2 - n - 12)$

$5(n+3)(n-4)$ ✱

71. $3m^2 + 37m + 12$

$(3m+1)(m+12)$ ✱

72. $3x^2 - 30x$

$3x(x-10)$ ✱

73. $x^2 + 8x + 16$

$(x+4)(x+4)$

$(x+4)^2$ ✱

74. $y^2 - 49$

$(y+7)(y-7)$ ✱

75. $9t^4 + 30t^3 + 25t^2$

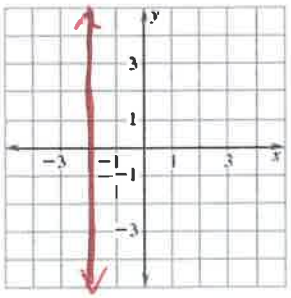
$t^2(9t^2 + 30t + 25)$

$t^2(3t+5)(3t+5)$

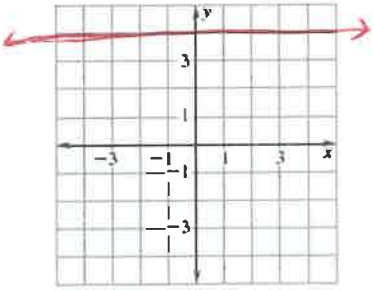
$t^2(3t+5)^2$ ✱

Graph each equation or inequality.

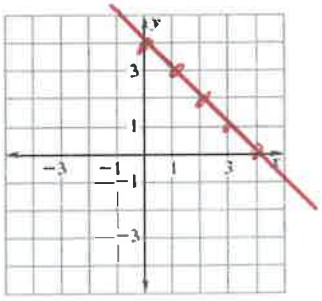
76. $x = -2$ vert line



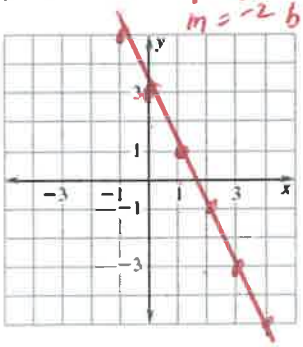
77. $y = 4$ horiz line



78. $y = -x + 4$ $m = -1$ $b = 4$



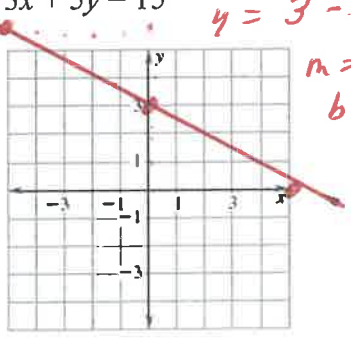
79. $y - 3 = -2x \rightarrow y = 3 - 2x$ $m = -2$ $b = 3$



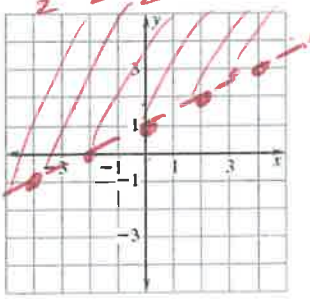
80. $3x + 5y = 15 \rightarrow 5y = 15 - 3x$

$y = 3 - \frac{3}{5}x$

$m = -\frac{3}{5}$ $b = 3$



81. $\frac{2y}{2} > \frac{x}{2} + \frac{2}{2} \rightarrow y > \frac{1}{2}x + 1$



dotted boundary (not included)

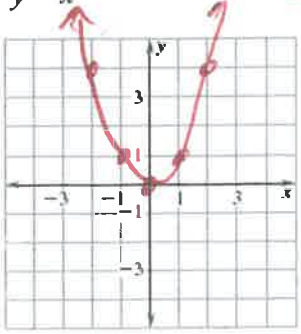
Test (0,0)

$0 \stackrel{?}{>} \frac{1}{2}(0) + 1$

$0 > 1$

no so "miss" (0,0)

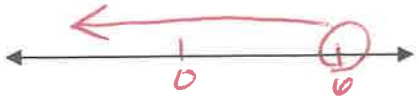
82. $y = x^2$



x	$y = x^2$
0	0
1	1
2	4
-1	1
-2	4

83. $\frac{5x}{5} < \frac{30}{5}$

$x < 6$



84. $-(x-4) \geq -3$

$-x + 4 \geq -3$

$-x \geq -7$

$x \leq 7$

Find the slope of the line that passes through each set of points. $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

85. $(-2, 2)$ and $(6, 6)$

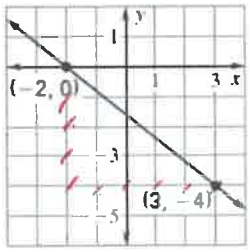
$m = \frac{6-2}{6--2} = \frac{4}{8} = \frac{1}{2}$ ✱

86. $(2, 10)$ and $(-2, 2)$

$m = \frac{2-10}{-2-2} = \frac{-8}{-4} = 2$ ✱

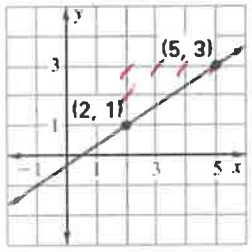
Use the graph to count and identify the slope of each line.

87.



$m = \frac{-4}{5}$ ✱

88.



$m = \frac{2}{3}$ ✱

Write an equation of the line in slope-intercept form. $y = mx + b$

89. slope: 5 y-intercept: -2

$y = 5x - 2$ ✱

90. Passes through (-2, 2) and (0, 4)

$b = 4$

$m = \frac{4-2}{0--2} = \frac{2}{2} = 1$

$y = 1x + 4$

91. Passes through (-4, 1); $m = 2$

$y = 1 + 2(x + 4)$

$= 1 + 2x + 8$

$y = 2x + 9$ ✱

92. Passes through (2, 8) and is horizontal.

$m = 0$

$y = 0x + 8$ ✱

$y = 8$ ✱

93. Passes through (4, -3) and is vertical.

m is undef

$x = 4$ ✱

94. Passes through (-3, 5) with a slope of $\frac{1}{4}$.

$y = \frac{1}{4}(x + 3) + 5$

$y = \frac{1}{4}x + \frac{3}{4} + 5 \rightarrow y = \frac{1}{4}x + \frac{23}{4}$ ✱

95. A line that is parallel to $3x + 4y = 7$ and goes through the point (-3, 8).

$m = \frac{-A}{B} = -\frac{3}{4}$ so $\| m = -\frac{3}{4}$

$y = 8 - \frac{3}{4}(x + 3)$

$y = 8 - \frac{3}{4}x - \frac{9}{4}$ $\rightarrow y = -\frac{3}{4}x + \frac{23}{4}$ ✱

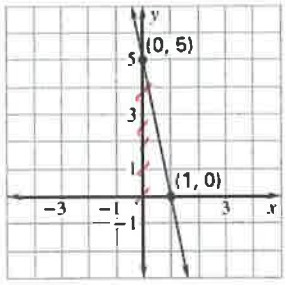
96. A line that is perpendicular to $y = 7 - 2(x + 4)$ and has a y-intercept of -3.

$m = -2$

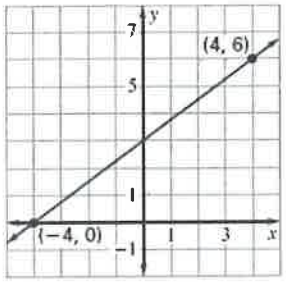
$\perp m = \frac{1}{2}$

$y = \frac{1}{2}x - 3$ ✱

97. Write an equation for the graphs in slope-intercept form.

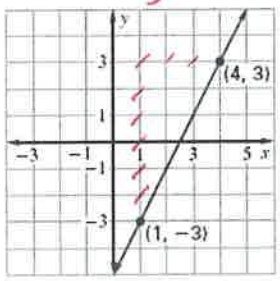


$y = 5 - 5x$ ✱



$y = 3 + \frac{3}{4}x$ ✱

$m = \frac{6}{3} = 2$

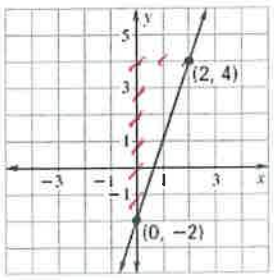


$y = -3 + 2(x-1)$

$= -3 + 2x - 2$

$y = -5 + 2x$ ✱

$m = \frac{6}{2} = 3$



$y = -2 + 3x$ ✱

98. $f(x) = 4x - 7$ and $g(x) = x^2 + 3x$

a. Find $f(3)$

$4(3) - 7 = 12 - 7 = 5$ ✱

b. Find $g(-8)$

$(-8)^2 + 3(-8) = 64 - 24 = 40$ ✱

c. Find $g(n-3)$

$(n-3)^2 + 3(n-3)$

$n^2 - 6n + 9 + 3n - 9 = n^2 - 3n$ ✱

d. If $f(x) = 6$, find x. ($f(x)$ = output)

$4x - 7 = 6$

$4x = 13$

$x = \frac{13}{4}$ ✱

e. Find $f(-5) + g(2)$

$4(-5) - 7 + 2^2 + 3(2)$

$-20 - 7 + 4 + 6$

-17 ✱

f. If $g(x) = 0$, find x. ($g(x)$ = output)

$x^2 + 3x = 0$

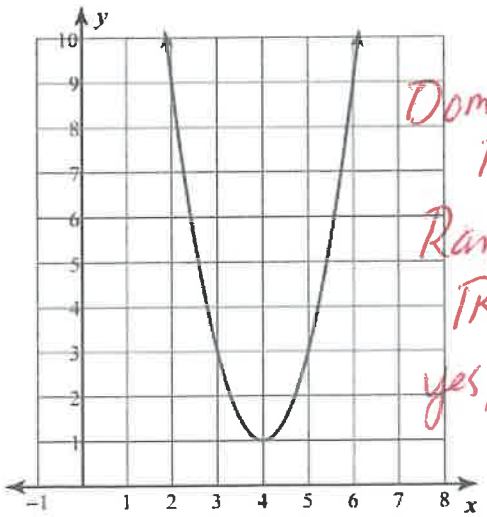
$x(x+3) = 0$

$x = 0$ ✱ | $x + 3 = 0$

$x = -3$ ✱

State the domain and range of each graph. Then state whether or not each graph represents a function.

99.

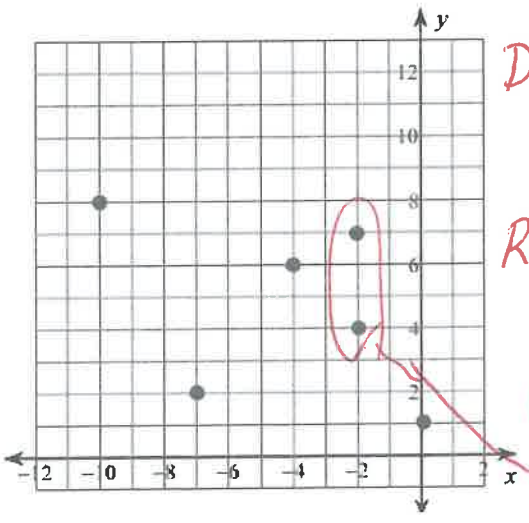


Domain: $\mathbb{R}$

Range: $\mathbb{R} \geq 1$

yes, fcn.

100.



D: {10, −7, 4, −2, 0}

R: {1, 2, 4, 6, 7, 8}

not a fcn b/c

$a^2+b^2=c^2$ makes rt △.

Determine whether or not the given numbers are possible measures for the sides of a right triangle.

101. 6, 8, and 10

$36+64 \stackrel{?}{=} 100$

$100 = 100$

yes

102. 11, 11, and 15

$121+121 \stackrel{?}{=} 225$

$242 \neq 225$

no

103. 2.7, 3.6, and 4.5

$2.7^2 + 3.6^2 \stackrel{?}{=} 4.5^2$

$7.29 + 12.96 \stackrel{?}{=} 20.25$

$20.25 = 20.25$

yes

104. 8, 15, and 17

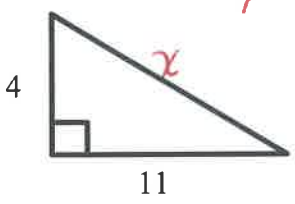
$64 + 225 \stackrel{?}{=} 289$

$289 = 289$

yes

Use the Pythagorean Theorem to find each missing side length. Give your answer in exact (radical) form and then use your calculator to approximate your answer to the nearest hundredth (two decimal places).

105.



$4^2+11^2=x^2$

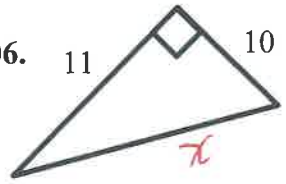
$16+121=x^2$

$137=x^2$

$x=\sqrt{137}$ (exact) ★

≈ 11.70 (approx) ★

106.



$11^2+10^2=x^2$

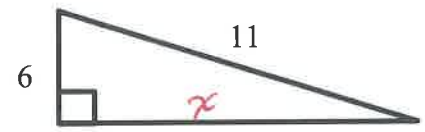
$121+100=x^2$

$221=x^2$

★ $\sqrt{221}=x$

★ $14.87=x$

107.



$6^2+x^2=11^2$

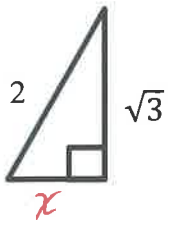
$36+x^2=121$

$x^2=85$

$x=\sqrt{85}$ ★

$=9.22$ ★

108.



$(\sqrt{3})^2+x^2=2^2$

$3+x^2=4$

$x^2=1$

$x=1$ ★

109. The students in one social studies class were asked how many brothers and sisters (siblings) they each have. The dot plot here shows the results.

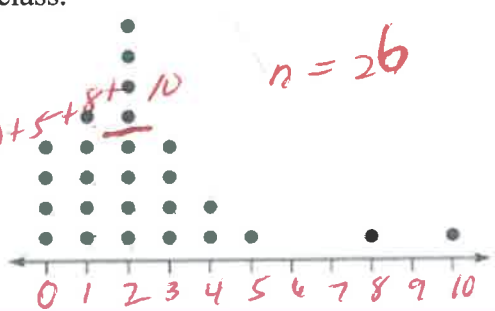
a. Find the mean median and mode number of siblings for this class.

mode = 2★ - most frequent response

median = (middle value) count in 13 = 2★

mean = $\frac{\text{total}}{26} = \frac{4(0)+5(1)+8(2)+4(3)+4(4)+1+8+10}{26}$

$= \frac{64}{26} \approx 2.46$ ★



n = 26

b. If a student were selected at random from this class, what is the probability that they have at least 4 siblings?

$$\frac{\#\text{ w/ 4 siblings}}{\#\text{ of ppl in class}} = \frac{2}{26} = \frac{1}{13}$$